Natalia Dellavalle

Rodney James

The World of Numbers

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The Appearance of the Fibonacci Sequence in Nature

Leonardo of Pisa, affectionately called Leonardo Fibonacci, first created the Fibonacci sequence in his 1205 work *Liber abaci.* In this influential work, he compiled the mathematical knowledge of his time, introduced Europe to the Hindu-Arabic numeral system, and produced the Fibonacci sequence with his unconventional "Rabbit Problem" (Vorobiev, 2002, p. 1).

Fibonacci first generated Fibonacci numbers to model the growth of a rabbit population. As stated in his reproduced manuscript published in 1228, Fibonacci wondered "how many pairs of rabbits can be bred from a single pair in one year" (Fibonacci, 1228). Fibonacci established reproductive restrictions on the rabbits to structure his problem. He specified that "each pair takes one month to mature," each reproductively capable pair produces a mixed pair of one male and one female rabbit who will reproduce with each other exclusively, and "no rabbits die during the course of the year" (Koshy, 2011, p. 4). Fibonacci also assumed the rabbits lived and reproduced in an isolated environment. Within these confines, he began to generate a sequence.

We begin with a pair of newborn rabbits on January 1st. They mature for one month, and the same pair of rabbits remains on February 1st. By March 1st, the pair has reproduced, and two pairs of rabbits now exist. On April 1st, the original pair has reproduced again while their progeny have matured, and three pairs of rabbits exist (Koshy, 2011, p. 4). Through these conditions, Fibonacci generated the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144… to find 144 pairs of rabbits alive come December (Koshy, 2011, p. 5). We consider these terms Fibonacci numbers, and may continue to generate terms of the Fibonacci sequence by adding the last two terms of the existing sequence to produce the next term (Vorobiev, 2002, p. 3).

Although the Fibonacci sequence misrepresents rabbit population growth, it correctly traces the ancestry of a male bee. Male bees develop from unfertilized eggs, while female bees develop from fertilized eggs. Therefore, each male bee has a mother, while each female bee has both a mother and father (Koshy, 2011, p. 26). To count a bee's ancestors in a given generation, we begin with one male bee. This male bee had one mother, and hence had one ancestor in the first generation removed. When we go back another generation, its mother had both a mother and father, so the bee had two ancestors in the second generation removed. We next regard the male bee's great-grandparents. The bee's grandmother had two parents, while the bee's grandfather had one parent, so the bee has three ancestors three generations removed. This actual condition generates the Fibonacci sequence (Koshy, 2011, p. 26).

In fact, the Fibonacci sequence persistently reappears in the natural world (Vorobiev, 2002, p. 2). Johannes Kepler first discovered the relationship between Fibonacci numbers and the Golden ratio. The ratio of any two consecutive Fibonacci numbers approaches the Golden Ratio as the Fibonacci numbers approach infinity (Livio 236). Mathematically, we denote this as: $\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \frac{1+\sqrt{5}}{2} = 1.61803398 \ldots$ (Adam, 2003, p. 213).

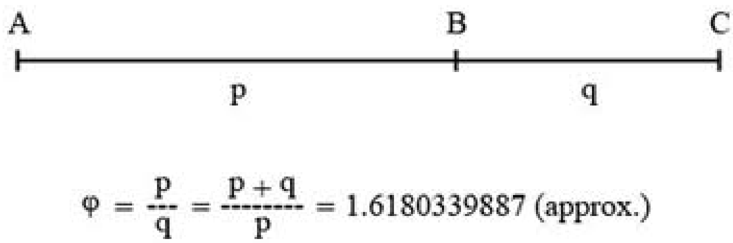

Figure 1: The Golden Ratio (Akhtaruzzaman, 2011)

As shown in Figure 1, a point B divides segment AC to construct the Golden Ratio. The ratio of the larger part AB to the smaller part BC equals the ratio of the whole AC to the larger part AB. If segment BC equals 1, AB must equal $\tau \approx 1.61803398$, creating the Golden Ratio of $\tau : 1$ (Adam, 2003, p. 214).

A “golden rectangle” with dimensions in the ratio $\tau : 1$ decomposes into a infinite series of smaller golden rectangles (Adam, 2003, p. 214). As shown in Figure 2, one may mark off a square from the largest golden rectangle, and a smaller golden rectangle remains (Adam, 2003, p. 215). This process continues indefinitely, and “corresponding points of each square spiral inward along a logarithmic or equiangular spiral” (Adam, 2003, p. 215). This “golden spiral,” as depicted in Figure 3, models the gnomonic growth of Nautilus shells (Adam, 2003, p. 215) and the descending path of peregrine falcons in pursuit of prey (Tucker, 2000, p. 10-13).

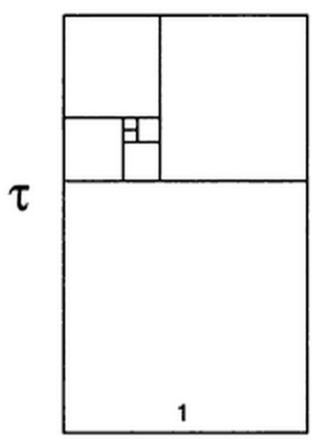

Figure 2: The Golden Rectangle (Dixon, 1981, p. 793)

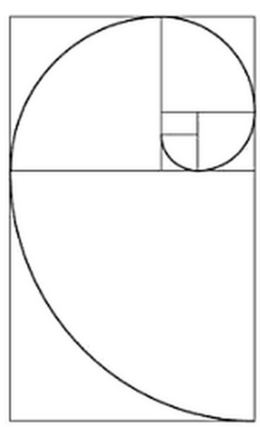

Figure 3: The Golden Spiral (Livio, 2009, p. 236)

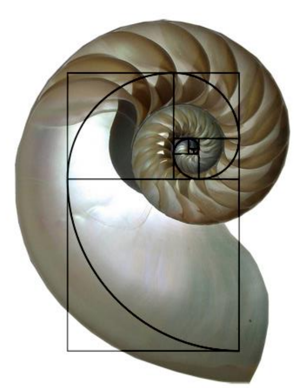

Figure 4: A Nautilus shell (Neuman, 2012)

In plants, the Fibonacci numbers readily appear in the number of petals of many flowers. For example, enchanter's nightshade has two petals; lilies, irises, and trilliums three; wild roses and buttercups five; delphiniums and cosmos eight; marigolds 13; asters 21; and daisies 34, 55, or 89 (Adam, 2003, p. 216; Koshy, 2011, p. 17).

The Fibonacci sequence also characterizes the phyllotaxis of plants—the "repetitive arrangement" of leaves, petals, seeds, florets, and branches of a plant (Adam, 2003, p. 216). "As a branch grows upward, it generates leaves at regular angular intervals" (Adam, 2003, p. 216). These intervals are generally not divisors of 360 degrees, because leaves prefer more space and sunlight; thus, they do not grow directly underneath one another when possible. Rather, the distribution of offshoots closely approximates irrational sections of one revolution around a stem (Adam, 2003, p. 216). For example, leaves sprout approximately every 2/5 of a revolution for oak, cherry, apple, apricot, holly, and plum; every 1/3 of a revolution for beech and hazel; every 3/8 of a revolution for poplar, rose, pear, and willow; and every 5/13 of a revolution for almond (Adam, 2003, p. 217; Coxeter, 1961, p. 169). Fibonacci numbers are the numerators and denominators of these phyllotactic ratios (Adam, 2003, p. 217).

Patterns of Fibonacci numbers recur in other plants as well. The exterior of a pineapple, for example, possesses "two interlocking families of helical spirals" (Stewart, 2011, p. 41). We quantify the number of spirals in each family with adjacent Fibonacci numbers eight and thirteen, so eight spirals wind clockwise and thirteen wind counterclockwise (Koshy, 2011, p. 24; Stewart, 2011, p. 41). A study of 2000 pineapples in the *1977 Yearbook of Science and the Future* affirmed this pattern. Artichokes and pinecones also share this arrangement, "with the number of spirals in the two directions often adjacent Fibonacci numbers," as do the seeds of a sunflower, although "the spirals are not helical, but lie in a plane" (Koshy, 2011, p. 22; Stewart, 2011, p. 42).

Although Fibonacci numbers present themselves abundantly in nature, no records show that Fibonacci even noticed the recursive property of the sequence; that is, he never realized one might add the last two terms of his sequence to reach the next. The Dutch mathematician Albert Girard first noticed this property nearly four centuries later, and defined the nth Fibonacci number, F_n, recursively as $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$ when the initial conditions $F_1 = F_2 = 1$ are met (Koshy, 2011, p. 6). Without observing the recursive property of his sequence, Fibonacci likely never foresaw another application of the sequence, let alone its ubiquitous, inherent presence. Although he "extended the material then known in geometry and trigonometry" (*Leonardo*, 2013), we now remember Fibonacci for the "sequence that bears his name" (Koshy, 2011, p. 5), one whose mysterious properties he did not discover nor enjoy.

The manmade Fibonacci sequence, declared a "fascinatingly prevalent tendency" by geometer H.M. Coxeter (1961, p. 10-11), manifests itself in the natural world so unpredictably. The Fibonacci sequence should arise frequently—architects considered τ when designing the Great Pyramid, the Parthenon, and the United Nations building; Leonardo DaVinci exhibited the proportion in his paintings; and we find the golden rectangle most aesthetically pleasing (Adam, 2003, p. 230)—but the great enigma lies in its ubiquitous, innate presence in our universe.

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