Student Research At CC

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Colorado College

Fearless Friday, September 12, 2008

Outline

Senior Capstone Projects, 2008

- Jes Coyle, "The Effect of Fire on Ponderosa Pine Forest Structure" (Steven Janke)
- Tra Ho, "Explicit Formulas for Real Hyperelliptic Curves of Genus 2 on the Projective Coordinate System" (Stefan Erickson)
- Matt Swanger, "A Fault Tolerable Content Addressable Network" (Jonathan Bredin)
- Jette Petersen, "The Initiation of Cancer via Two Mutations" (David Brown)
- Andrea Buchwald, "The Unit Group of Real Biquadratic Number Fields" (Stefan Erickson)

Math 345: Research in Mathematics - Spring, 2008

- Andrew Bean, "The Mathematics of Phylogenetic Trees"
- Melissa Miller, "Numerical Frieze Patterns"
- Kayla Valvo, "P^{th} Roots of Unity and a Norm Function"

Summer 2008

- Samuel Zemedkun, "Hyperelliptic Curve Cryptography" (Stefan Erickson, Venture Fund for Faculty-Student Collaborative Grants)
- Molly Moran, "Polynomial Invariants" (Bob Pelayo, Venture Grant)
- Andrew Bean and Laura McQuaid, "Phylogenetics: Data Analysis and Mathematica Discussion" (Amelia Taylor and Ralph Bertrand, NSF-UBM)
- Cory Beyer and Quintana Baker, "Demographics and Population Dynamics of the Flammulated Owl" (Steven Janke and Brian Linkhart, NSF-UBM)
- Noah Brostoff and Laura Johnson, "The GASP Phenotype: A Mathematical and Experimental Study of Evolution in Bacteria" (David Brown and Phoebe Lostroh, NSF-UBM)

Other Opportunities

- Molly Moran and Sara Wolff, Summer Program for Women in Mathematics (SPWM) at George Washington University (women completing junior year, `http://www.gwu.edu/ spwm/`).
  Deadline: February 27, 2009
- Other NSF Research for Undergraduates opportunities.
  `http://www.nsf.gov/crssprgm/reu/reu_search.cfm`
  Deadlines are generally January - March
- Kayla Valvo, FBI Honors Internship (`http://www.fbijobs.gov/231.asp`),
  NSA Directors Summer Program (`http://www.nsa.gov/CAREERS/students_1.cfm?#dsp`),
  Deadlines are in October!

SET™ Game

Consists of a deck of cards.

Exactly one card for each combination.

Color	Number	Shape	Shade
red	1	oval	open
green	2	diamond	solid
purple	3	squiggle	striped

A *SET* is a collection of 3 cards such that for every parameter the cards are all the same or all different.

SET™Game

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A *SET* is a collection of 3 cards such that for every parameter the cards are all the same or all different.

Let $R = k[x_1, \ldots, x_{81}]$ with each variable corresponding to a card in the SET™deck.

Set I to be the ideal generated by $x_i x_j x_k$ where x_i, x_j and x_k correspond to 3 cards that form a SET™.

SET[TM] Game: The Question

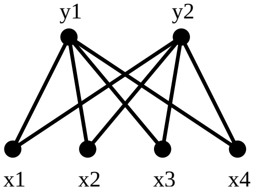
Q: What is the structure of the ideal I? There are many properties we can experiment with computationally to see if I satisfies them in low dimensions and look for good reasons and proofs that they work for all dimensions.

Prereq: Linear Algebra (MA220). A willingness to learn about a few new structures and interest in playing with a computer program that is new to you. Abstract Algebra 1 (MA321) is a plus.

Free Resolutions of Graph Ideals: An Example

Theorem (Visscher, Coyle)

If I is the ideal of the hypergraph generated by the maximal cliques of a complete multi-partite graph, then the cellular resolution is a minimal free resolution.



$$0 \longrightarrow R^{n_2} \xrightarrow{\phi_2} R^{n_1} \xrightarrow{\phi_1} R^{n} \xrightarrow{\phi} M \longrightarrow 0$$

Free Resolutions of Graph Ideals

Q: Our understanding of and techniques for studying graph ideals has recently bloomed. However, there is still much to be learned. Including simply extending the work of Visscher and Coyle.

Prereq: Linear Algebra and Abstract Algebra I (MA321). Abstract Algebra II (MA322) and Graph Theory (MA325) are a plus.

Structure of Projections of Rational Curves: Examples/Exploration

Q: When is a monomial ideal the initial ideal of a prime ideal? What generic initial ideals occur as the initial ideal of a prime ideal? It seems like an interesting class of gins appear as the initial ideal of certain projections of rational curves. These are easy to define and play with computationally. What other projections can we from? Which other gins can we get? How might we prove the structure?

Prereq: Linear Algebra and Abstract Algebra I (MA321). Geometry (MA300) is a plus. A willingness to play with a computing program that is new to you.

Phylogenetics

Q: I have several questions to work on regarding phylogenetics. Most of my current projects are computational/computer and/or statisically based, although I also work on problems related to algebra.

Prereq: Variable. Probability (MA313) and Mathematical Statistics (MA417) for statiscally based questions. Computing experience a strong plus. Linear Algebra (MA220), Abstract Algebra (MA321), and some experience with statistics and computing for algebra based problems.

Outline

A History Project, Quadrature of the Lune

Definition

A figure A is "quadrable" if one can construct with a ruler and compass a square that has the exact area as A. If this can be done we say that we have "squared" A.

Hippocrates of Chios (460 - 380 BC) was able to demonstrate the quadrature of several lunes, in fact his demonstrations are believed to be the earliest "modern" proofs we have.

In the early 1770's Euler showed that three more lunes were "squarable" which gave us a grand total of five quadrable lunes.

In the early 20th century two Russian mathematians proved independently that these five were the ONLY quadrable lunes.

A Reading Project, Reduced Rings

In the survey paper "The Minimal Prime Spectrum of a Reduced Ring," (*Illinois Journal of Mathematics*, 1983) Eben Matlis collected his and others' results on this important class of rings with zero divisors.

Definition

A ring R is *reduced* if $a^2 = 0$, $a \in R$, implies $a = 0$.

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A "Find Some Examples" Project

Theorem

A semigroup S is a semigroup of divisibility of a semihereditary Bézout ring if and only if it is a semihereditary B-semigroup.

Definition

A *D-semigroup* is a commutative distributive monoid under the natural partial order with 0 and the unique invertible element 1. A *Bézout semigroup* S (in short a *B-semigroup*) is a *hypernormal* D-semigroup S; that is, for any $x, y, d = x \wedge y \in S$ and $dx_1 = x$ there is y_1 with $x_1 \wedge y_1 = 1$, $dy_1 = y$. A B-semigroup S is called *semihereditary* if for every element $a \in S$ there is an idempotent $e \in S$ such that eS is the annihilator $a^{\perp} = \{x \in S \mid ax = 0\}$ of a.

Matching Strategic Agents Online

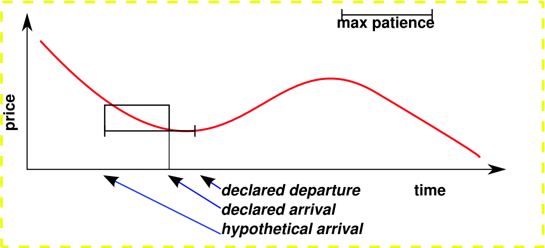
- 2 agent types
  - publically known
  - buyers, sellers
  - employers, employees
  - women, men
- Private revelation
  - value, cost
  - match window

Applications - Matching Strategic Agents Online

- Auctions
- Job scheduling
- Matching service

Examples: Matching Strategic Agents Online

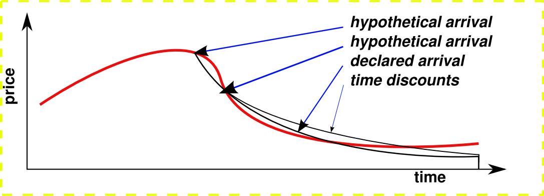
- Patience threshold, k
- Price: max-price (t, d-k)
- Trade reduction



Examples: Matching Strategic Agents Online

- Time discount?
- $u = (x - v)e^{-\gamma(t-a)}$
- Approximately strategy-proof

-- Time discount?



Desirderata - Matching Strategic Agents Online

- Budget-balanced
- Online
- Strategy-proof

Matching Strategic Agents Online

Q: Are there (approximate) strategy-proof methods to match self-interested agents? Can we match agents with discrete preferences?

Prereq: Data Structures (CP222). Some interest in economics would be nice.

When: School year or summer.