Introduction

The Cosmic Microwave Background (CMB) is a sea of microwave photons that fills the universe. The CMB is very nearly isotropic, and the energy distribution of photons is nearly perfectly described by the blackbody spectrum. The current average temperature of a CMB photon is 2.755 K, and there are about 410,700,000 photons in any given cubic meter of space. The discovery of these photons by Arno Penzias and Robert Wilson in 1964 constitute the first evidence that the universe once existed in a hot, dense state.

Causes

Since the universe is expanding, it must have been denser in the past. Due to high energy density just after the Big Bang, the universe was once hot enough that electrons could not bind to protons to form atoms. Atomic reaction governing the ionization of the early universe

\[ p + e^- \leftrightarrow H + \gamma \]

The above atomic reaction describes the formation or ionization of hydrogen through the release or uptake of a photon and has an ionization energy \( Q = 13.6 \) eV.

Decoupling

At temperatures with energy above this ionization energy, hydrogen ionizes and electrons are free to interact with photons through Compton and Thomson scattering, binding them together. The minute the universe cooled enough for atoms to form is called Recombination, which trapped the electrons and released the photons as the CMB in an event called Decoupling.

\[ n_x(p)dp = g_x \frac{4\pi}{h^3} \frac{p^2 dp}{e(E - \mu_x)/kT \pm 1} \]

\[ n_\gamma(f)df = \frac{8\pi}{c^3} \frac{f^2 df}{ehf/kT - 1} \]

Momentum-dependent particle density of massive particles, and frequency dependent blackbody spectrum for photons

Recombination is mathematically defined as the moment when only half the atoms in the universe are ionized. This is tracked using the fractional ionization \( X \), or the ratio between the number of charged particles to the total number of particles in the universe, and recombination is when \( X = \frac{1}{2} \). Using the above equations, we can define \( X \) in terms of temperature \( T \), the ratio of photons to baryons \( \eta \), and the ionization energy.

\[ X \equiv \frac{n_p}{n_p + n_\mu} = \frac{1 - X}{X} = n_p \left( \frac{m_e kT}{2\pi \hbar^2} \right)^{-3/2} e^{Q/kT} \]

\[ \frac{1 - X}{X^2} = 3.84 \eta \left( \frac{kT}{m_e c^2} \right)^{3/2} e^{Q/kT} \]

\[ T_{\text{rec}} = 3760 \text{ K} \]

Abbreviated derivation to find the temperature of Recombination

Once the universe’s expansion caused it to cool to this temperature, recombination then, consequently, decoupling occurred.

Recombination

Assuming the universe is in thermal, kinetic, and chemical equilibrium, we can use statistical mechanics to determine the temperature of Recombination. We use Fermi–Dirac statistics to determine the number density of massive particles \( n_x \), and the blackbody distribution to determine the number density of photons \( n_\gamma \).

References