Deriving PV=nRT

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Introduction
Did you know?
Only about a hundred substances are gases at room temperature
Did you know?
Gas particles move at over 1,000 miles per hour (450 meters per second)

Assumptions
- Low Density:
  - Volume of molecule is much less than that of the room
  - Molecules are point particles
- Collisions are elastic

Acknowledgments
Thank you to Kristine Lang for providing resources for this project. Books used:
Schroeder, Daniel; *An Introduction to Thermal Physics*
Stein, James D.; *Cosmic numbers: the numbers that define our universe*
Dugdale, J.S.; *Entropy and Low Temperature Physics*

Starting with one molecule
Consider it hitting the wall of a container. We will only consider momentum change in one direction before scaling up to three dimensions for a real particle.

\[
\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta m \vec{v}}{\Delta t} = \frac{m \Delta \vec{v}}{\Delta t} = \frac{2m v_x}{\Delta t}
\]

and

\[
v_x = \frac{2l}{\Delta t} \quad \text{so,} \quad \Delta t = \frac{2l}{v_x}
\]

with some algebra, we can combine these equations:

\[
F = \frac{mv_x^2}{l}
\]

Pressure is a force over some area, so we can write \( P_1 \) to denote the pressure from one molecule

\[
\frac{F_1}{A} = P_1
\]

we scale this up to \( N \) particles by writing:

\[
P_N = N \frac{mv_x^2}{l^3} \quad \text{eq 1}
\]

Finally
Combining equation 1 and equation 2 and knowing \( V \) is Volume, \( V \), we can write:

\[
P_N = N \frac{k_B T}{V}
\]

Which is equivalent to \( PV = nRT \)

To multiple dimensions
To scale up from 1 dimension to three, we can recognize that we arbitrarily chose a direction for our velocity, and that there is nothing special about the \( x \)-direction. So, the average velocity in all directions are equal.

\[
\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2
\]

and

\[
\bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2
\]

so,

\[
\bar{v}^2 = 3 \bar{v}_x^2
\]

Lastly
We bring it home with an application of Maxwell-Boltzmann distribution. Knowing:

\[
KE = \frac{1}{2} m v^2 \quad \text{and} \quad KE = \frac{3}{2} k_B T
\]

we write,

\[
\frac{1}{2} m v^2 = \frac{3}{2} k_B T \quad \text{and} \quad m v^2 = 3 k_B T \quad \text{eq 2}
\]