Reversible First Stage Rockets

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The Rocket Formula

\[ P(t) = mv \]

\[ P(t + dt) = (m + dm)(v + dv) - dm(v - v_{ex}) \]

\[ dP = P(t + dt) - P(t) = mdv + v_{ex}dm = 0 \]

\[ m dv = -v_{ex}dm \quad \text{Divide by } dt \]

\[ \dot{m} = -\dot{v}v_{ex} \]

\[ F_{\text{thrust}} = -\dot{r}v_{ex} \quad \text{Mass flow rate } = \dot{m} \]

Specific impulse for a rocket:

\[ I_{sp} = \frac{F_{\text{thrust}}}{mg_0} \]

\[ m dv = -v_{ex}dm \quad \text{Divide by } m \]

\[ \dot{v} = \frac{v_{ex}dm}{m} \]

\[ v(t) = v_0 + v_e \ln \frac{M_0}{M(t)} \]

\[ M_f = M_0 - M_f = 2.157 \times 10^5 \text{kg} \]

\[ M_{prop} = M_{prop} - M_{loss} = 1.571 \times 10^5 \text{kg} \]

Launch

How much mass is lost? What is the mass loss?

\[ v_{ex} = (9.8 \text{ m/s}^2)(312) = 3058 \text{ m/s} \]

\[ \dot{m} = F_{\text{thrust}} = -\frac{7.543 \times 10^6 \text{ kg}}{-3058 \text{ m/s}} = -2.467 \times 10^3 \text{kg/s} \]

\[ M_f = (5.135 \times 10^5 \text{kg})e^{-\frac{3058 \text{ m/s}}{8030 \text{ m/s}}} = 2.978 \times 10^3 \text{kg} \]

Flip Maneuver

How much time does it take for the first stage to descend? We assume this is the time it takes for the first stage to reorient itself.

\[ \theta = 47^\circ \]

\[ v_f = v_0 + \frac{2g(y_f - y_0)}{t} \]

\[ v_f = v_0 + gt \]

\[ t = \frac{-v_0}{g} = 124 \text{ sec} \]

What is the center of mass? Let us think of the first stage in two parts: a uniform hollow and solid cylinder stacked on one another. Assume cold gas thrusters that provide the force to flip are at a height of 35.8 meters and that the height of separation is 10.23 meters.

On December 22nd 2015, the Falcon 9 Orbcomm-2 first stage lands successfully. This is the first ever orbital-class rocket landing. From the video and flight logs, we can gather specifications about the first stage.

Flight Specifications

- Thrust @ vac = 7,543 MN
- Total Mass = 5,135 x 10^3 kg
- First Stage Mass = 3,975 x 10^3 kg
- Propellant Mass = 3,728 x 10^3 kg
- Initial velocity = 1666 m/s
- Motor = 75.4 km
- Orbit = 620 km x 660 km x 42°
- Merlin 1D mass (first stage) = 4230 kg

| Source: SpaceX (web, launch video, flight logs) |