Cosmic Microwave Background: Recombination and Decoupling

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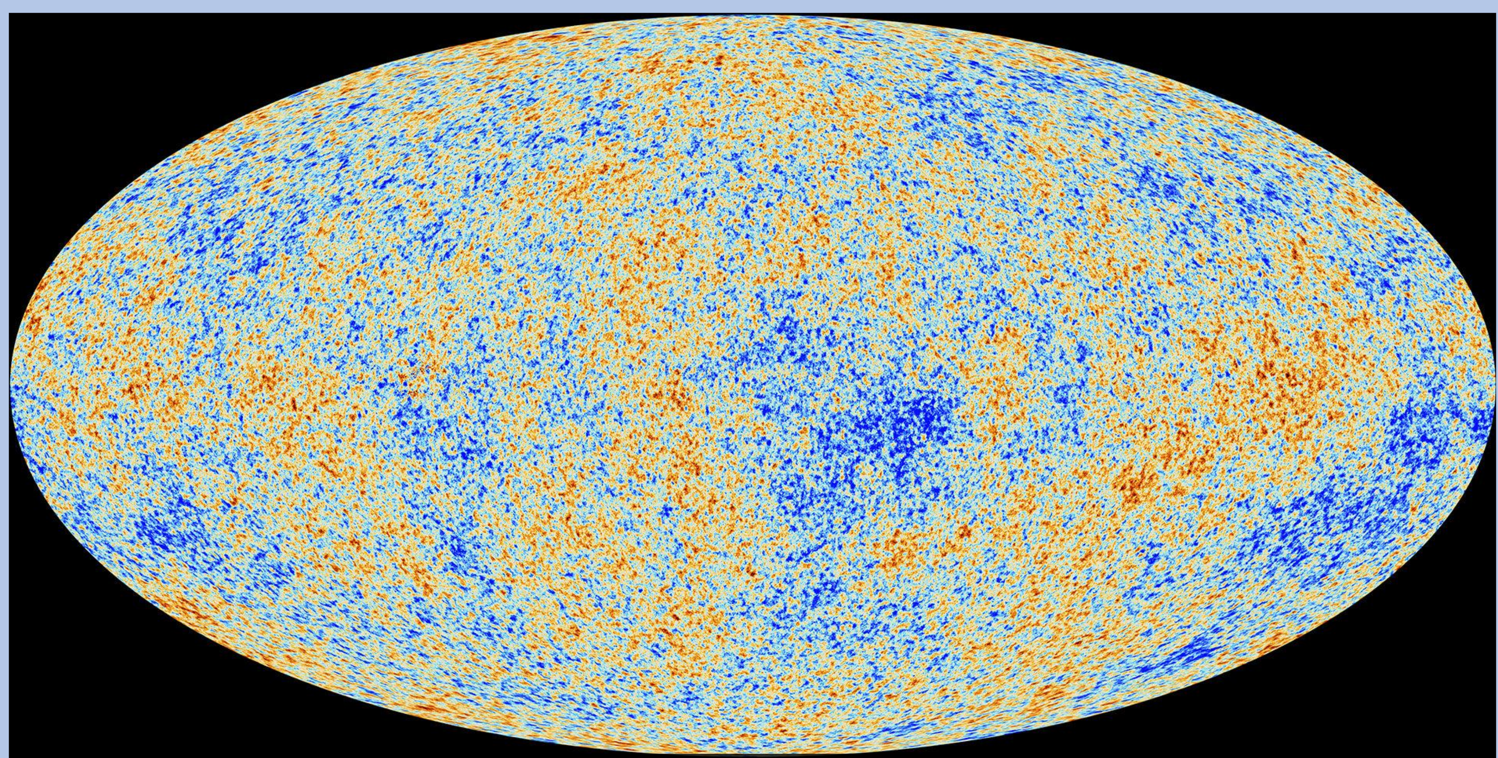
Introduction

The Cosmic Microwave Background (CMB) is a sea of microwave photons that fills the universe.



The CMB. Credit: ESA Planck-III

The CMB is very nearly isotropic, and the energy distribution of photons is nearly perfectly described by the blackbody spectrum. The current average temperature of a CMB photon is 2.755 K, and there are about 410,700,000 photons in any given cubic meter of space. The discovery of these photons by Arno Penzias and Robert Wilson in 1964 constitute the first evidence that the universe once existed in a hot, dense state.

Causes

Since the universe is expanding, it must have been denser in the past. Due to high energy density just after the Big Bang, the universe was once hot enough that electrons could not bind to protons to form atoms.

$$p + e^- \rightleftharpoons H + \gamma$$

Atomic reaction governing the ionization of the early universe

The above atomic reaction describes the formation or ionization of hydrogen through the release or uptake of a photon and has an ionization energy $Q = 13.6$ eV.

At temperatures with energy above this ionization energy, hydrogen ionizes and electrons are free to interact with photons through Compton and Thomson scattering, binding them together. The moment the universe cooled enough for atoms to form is called Recombination, which trapped the electrons and released the photons as the CMB in an event called Decoupling.

Decoupling

Decoupling is mathematically defined as the moment when the rate at which photons scatter is equal to the expansion rate of the universe. This defines when a photon is no longer likely to be scattered, “releasing” it as the CMB.

$$\Gamma = \frac{n_{bary}\sigma_e c}{a^3} = H = \frac{2.1 \times 10^{-20}\,\mathrm{s}^{-1}}{a^2}$$

Definition of Decoupling, where Γ is the scattering rate, n_{bary} is the number density of baryons and is equal to n_e, σ_e is the cross-sectional area of an electron, H is the expansion rate of the universe, and a is the dimensionless scalefactor

This uses an electron density based on a fully ionized universe, so predicts that Decoupling wouldn’t happen until just 22mya, after the extinction of the dinosaurs. Recombination rapidly decreases the density of electrons, so Decoupling and Recombination happen around the same time, ~380,000 years after the Big Bang.

Recombination

Assuming the universe is in thermal, kinetic, and chemical equilibrium, we can use statistical mechanics to determine the temperature of Recombination. We use Fermi-Dirac statistics to determine the number density of massive particles n_x, and the blackbody distribution to determine the number density of photons n_γ.

$$n_x(p)dp = g_x \frac{4\pi}{h^3} \frac{p^2 dp}{e^{(E-\mu_x)/kT} \pm 1}$$

$$n_\gamma(f)df = \frac{8\pi}{c^3} \frac{f^2 df}{e^{hf/kT} - 1}$$

Momentum-dependent particle density of massive particles, and frequency dependent blackbody spectrum for photons

Recombination is mathematically defined as the moment when only half the atoms in the universe are ionized. This is tracked using the fractional ionization X, or the ratio between the number of charged particles to the total number of paticles in the universe, and recombination is when $X = {}^1/_2$. Using the above equations, we can define X in terms of temperature T, the ratio of photons to baryons η, and the ionization energy.

$$X \equiv \frac{n_p}{n_H + n_p} \Rightarrow \frac{1-X}{X} = n_p \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} e^{Q/kT}$$

$$\Rightarrow \frac{1-X}{X^2} = 3.84\eta \left(\frac{kT}{m_e c^2}\right)^{3/2} e^{Q/kT}$$

$$\Rightarrow T_{rec} = 3760\,\mathrm{K}$$

Abbreviated derivation to find the temperature of Recombination

Once the universe’s expansion caused it to cool to this temperature, recombination then, consequently, decoupling occurred.

References

[1] Bradley Carrol and Dale Ostlie. *An Introduction to Modern Astrophysics*. New York: Pearson, 1996.

[2] William Heacox. *The Expanding Universe: An Introduction to Relativistic Cosmology*. Upper Saddle River, NJ: Pearson Eductation, Inc., 2018.

[3] Barbara Ryden. *An Introduction to Cosmology*. Cambridge, UK: Cambridge University Press, 2017.