Coriolis Force on Tropical Cyclone Formation

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Inertial Forces

The Coriolis Force is an inertial force that originates from examining motion in a rotating reference frame. Newton's Second Law holds well in cases where both reference frames are inertial, or non-accelerating. Newton's Second does not hold well in cases where one reference frame is accelerating with respect to another. An example of this is a train car in motion, with a ball being thrown inside of the train car. The inertial frame is anchored to the ground, and the non-inertial frame is a frame fixed in the car that is moving relative to the inertial frame with a given velocity and acceleration. When the ball's motion is examined and measured relative to the accelerating frame, Newton's Second looks a bit different.

$$m\ddot{\boldsymbol{r}}_{\boldsymbol{o}} = \boldsymbol{F}$$

Ball's motion relative to the inertial frame

$$\dot{\boldsymbol{r}}_{\boldsymbol{o}} = \dot{\boldsymbol{r}} + \boldsymbol{V}$$

Total velocity of the ball (relative to ground = relative to car + car)

$$\ddot{\boldsymbol{r}} = \ddot{\boldsymbol{r}}_{\boldsymbol{o}} - \boldsymbol{A}$$

Previous equation differentiated

$$m\ddot{\boldsymbol{r}} = \boldsymbol{F} - m\boldsymbol{A}$$

Newton's Second in the non-inertial frame

Note the addition of the extra term, the inertial force. We experience inertial forces often, like when an airplane takes off and we feel ourselves pushed back into the seat, or when a car goes around a tight curve and centrifugal force pushes passengers outwards. The Coriolis Force is an inertial force, and must be included when we use Newton's Second in a rotating frame.

$$m\left(\frac{d^2\boldsymbol{r}}{dt^2}\right)_{S_o} = \boldsymbol{F}$$

Start with this, a particle with mass in some position in the inertial frame.

$$\left(\frac{d\boldsymbol{Q}}{dt}\right)_{S_o} = \left(\frac{d\boldsymbol{Q}}{dt}\right)_{S} + \Omega \times \boldsymbol{Q}$$

This equation relates derivatives of vectors in the inertial frame to the corresponding derivative in the rotating frame. We can use it on the previous equation to solve for Newton's Second in a rotating frame. Note that Omega here is the angular velocity of the earth.

$$m\ddot{\boldsymbol{r}} = \boldsymbol{F} + [2m\dot{\boldsymbol{r}} \times \boldsymbol{\Omega}] + [m(\boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{\Omega}]$$

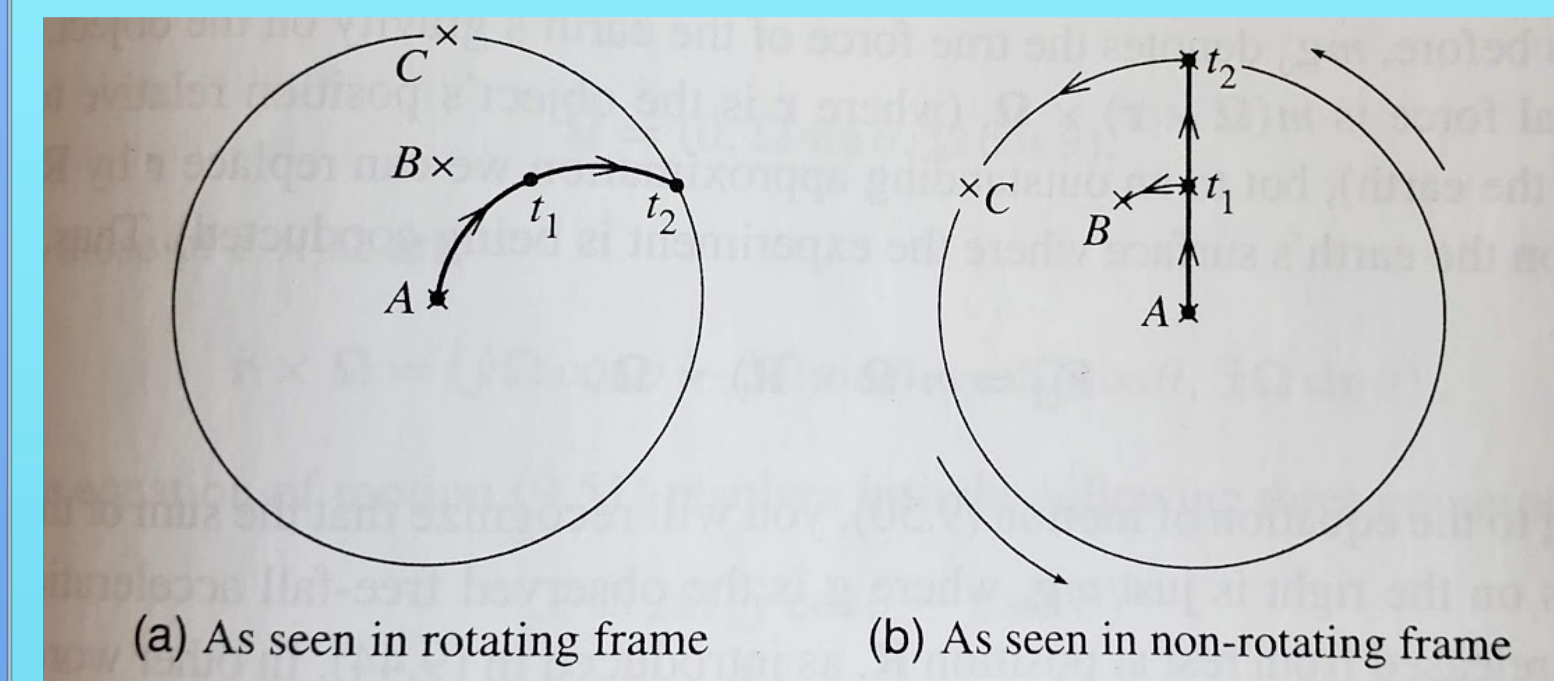
We wind up with this equation, with two additional inertial force values. The first is the Coriolis Force, the other is the centrifugal force.

$$\boldsymbol{F}_{cor} = [2m\boldsymbol{v} \times \boldsymbol{\Omega}]$$

This is the Coriolis Force, strongly dependent on the velocity of the particle and the angular velocity of the earth.

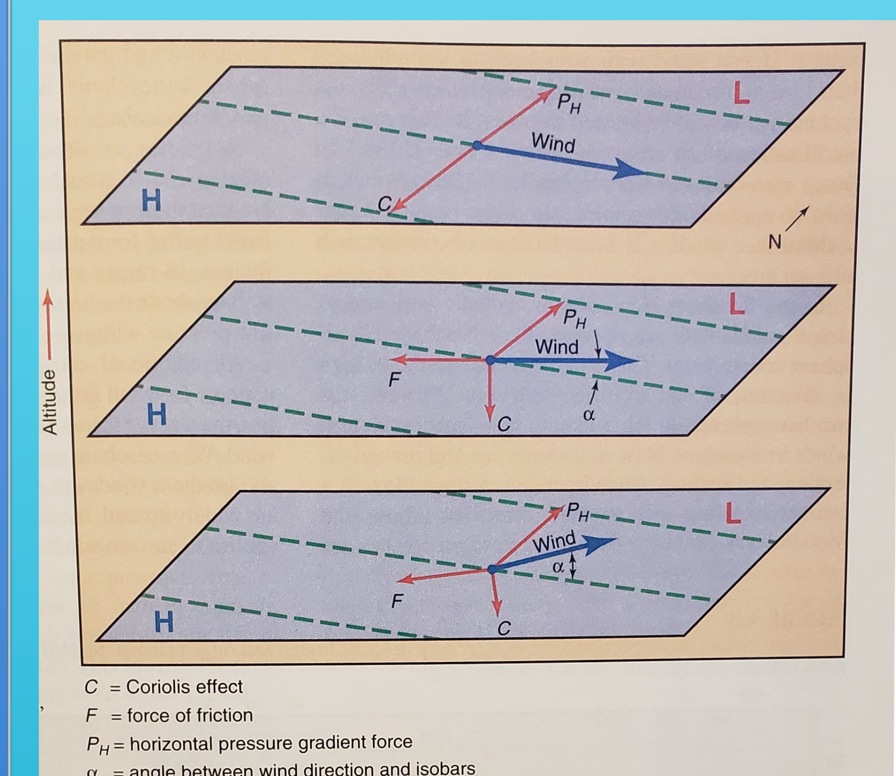
Coriolis Force

The Coriolis Force deflects particles perpendicular to their path of travel, with a direction given by the right hand rule. In the northern hemisphere, this direction is always to the right of the velocity vector of the moving particle.



How the seen path of a particle varies depending on what frame the observer is in.

The Coriolis Force is responsible for the formation of cyclones, as long as the air parcel that is moving is close enough to the surface to be affected by friction.



In this image, the wind is traveling towards the low pressure center, with the Coriolis Force balancing the pressure gradient force at high altitudes. At lower altitudes, friction slows the air parcel down, resulting in a lower Coriolis Force, curving the wind into the low pressure center. This is effectively how cyclonic motion originates in most cases.

Coriolis Parameter

The Coriolis Parameter is what meteorologists use to quantify the effects of the Coriolis Force on weather, based upon latitude.

$$f = 2\Omega \sin\varphi$$

Where phi is the latitude of the particle. Evidently, the deflection due to the Coriolis Force is greatest at the poles and least at the equator, and the Coriolis parameter reflects this by being 0 at the equator and 1 at the poles.

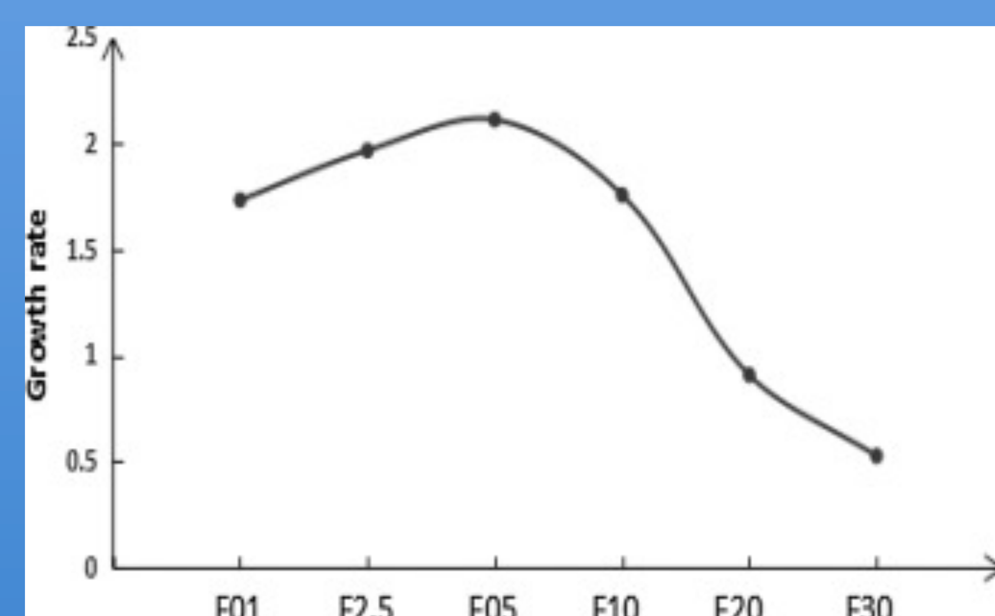
$$\frac{\partial\zeta}{\partial t} = -fD$$

One particular relationship examined by Deng et al (2018) is the reliance of the change in relative vorticity over time on the divergence of the air and the Coriolis parameter – for some given low level convergence, large Coriolis parameter causes greater acceleration of vorticity, favoring stronger tropical cyclone development.

$$\frac{\partial D}{\partial t} = f\zeta - \nabla^2\phi$$

Concurrently ,the other relationship examined determined that a greater Coriolis parameter would lead to a greater divergence over time, which would slow tropical cyclone development. Here, the rightmost value is geopotential, the variance in the potential of earth's gravity with height.

Evidently, we need some balance between factors, and Deng et al found that the Coriolis parameter controls the timing of rapid intensification and overall size of a given tropical cyclone. In essence, an optimal Coriolis parameter would yield a cyclone that develops faster and is larger, but is not necessarily more intense.



Growth rate of a tropical cylone peaks at about 5 degrees latitude. If you're interested in more information, scan the QR code for Deng et al's paper.

Acknowledgments

Thanks to everyone in the physics department for helping me out with this seminar!

Image References:
Image #1: Taylor
Image #2: Moran and Morgan
Image #3 : Deng et al 23018)